

## **The Knowledge Quartet: a framework for analysing and developing mathematics teaching**

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**Abstract.** *This paper describes a framework for mathematics lesson observation, the ‘Knowledge Quartet’, and the purposes for which it was developed. A grounded theory approach to the analysis of many hours of classroom mathematics teaching led to the emergence of the framework, with four broad dimensions, through which the mathematics-related knowledge of the teacher participants could be observed in practice. This paper describes how each of these dimensions is characterised, and analyses one lesson, showing how each dimension of the Quartet can be identified in it. The paper concludes by outlining recent developments in the use of the Knowledge Quartet.*

**Key words:** mathematics teaching, teacher knowledge, teacher education, Knowledge Quartet.

### **Introduction**

This paper concerns a framework for the analysis of mathematics teaching – the Knowledge Quartet – which was first developed at the University of Cambridge in the years 2002–4. Since then, the Knowledge Quartet has been applied in several research and teacher education contexts, and the framework has been further refined and developed as a consequence. The paper begins with a description of the research study which led to the emergence of the Knowledge Quartet, and how key elements of the theory are conceptualised. It proceeds to an analysis of one lesson through the lens of the Knowledge Quartet, and concludes with a discussion of some of the ways in which the framework has been used and developed further.

### **Developing the Knowledge Quartet**

#### ***Context and purpose of the research***

In the UK, the majority of prospective, ‘trainee’ teachers are graduates who follow a one-year program leading to a Postgraduate Certificate in Education (PGCE) in a university<sup>1</sup> education department. Over half of the PGCE year is spent teaching in

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<sup>1</sup> It should be noted, however, that the government now actively promotes a range of workplace-based alternatives (such as ‘School Direct’) to the PGCE. These are effectively located in notions of apprenticeship, and offer little interaction with university-based teacher educators.

schools under the guidance of a school-based mentor, or ‘cooperating teacher’. Placement lesson observation is normally followed by a review meeting between the cooperating teacher and the student-teacher. On occasion, a university-based tutor will participate in the observation and the review. The evidence indicates that these mentor/trainee lesson review meetings typically focus heavily on organisational features of the lesson, with very little attention to the mathematical content of mathematics lessons (Borko & Mayfield, 1995; Strong & Baron, 2004).

The purpose of the research from which the Knowledge Quartet emerged was to develop an empirically-based conceptual framework for lesson review discussions *with a focus on the mathematics* content of the lesson, and the role of the trainee’s mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK). In order to be a useful tool for those who would use it in the context of *practicum* placements, such a framework would need to capture a number of important ideas and factors about mathematics content knowledge in relation to teaching, within a small number of conceptual categories, with a set of easily-remembered labels for those categories.

The research reported in this paper was undertaken in collaboration with Cambridge SKIMA colleagues Peter Huckstep, Anne Thwaites, Fay Turner and Jane Warwick. I frequently, and automatically, use the pronoun ‘we’ in this text in recognition of their contribution.

### ***Method***

The participants in the first, theory-generating phase of the study were enrolled on a one-year PGCE course in which each of the 149 trainees specialised either on the Early Years (pupil ages 3–8) or the Primary Years (ages 7–11). Six trainees from each of these groups were chosen for observation during their final school placement. The six were chosen to reflect a range of outcomes of a subject-knowledge audit administered three months earlier. Two mathematics lessons taught by each of these trainees were observed and videotaped, i.e. 24 lessons in total. The trainees were asked to provide a copy of their planning for the observed lesson. As soon as possible after the lesson the observer/researcher wrote a succinct account of what had happened in the lesson, so that a reader might immediately be able to contextualise subsequent discussion of any events within it. These ‘descriptive synopses’ were typically written from memory and field notes, with occasional reference to the videotape if necessary.

From that point, we took a grounded approach to the data for the purpose of generating theory (Glaser & Strauss, 1967). In particular, we identified in the videotaped lessons aspects of trainees' actions in the classroom that seemed to be significant in the limited sense that it could be construed to be informed by a trainee's mathematics subject matter knowledge or their mathematical pedagogical knowledge. We realised later that most of these significant actions related to choices made by the trainee, in their planning or more spontaneously. Each was provisionally assigned an 'invented' code. These were grounded in particular moments or episodes in the tapes. This provisional set of codes was rationalised and reduced (e.g. eliminating duplicate codes and marginal events) by negotiation and agreement in the research team. The 17 codes generated by this inductive process are itemised later in this chapter. The name assigned to each code is intended to be indicative of the type of issue identified by it: for example, the code *adheres to textbook* (AT) was applied when a lesson followed a textbook script with little or no deviation, or when a set of exercises was 'lifted' from a textbook, or other published resource, sometimes with problematic consequences.

Equipped with this set of codes, we revisited each lesson in turn and, after further intensive study of the tapes, elaborated each descriptive synopsis into an analytical account of the lesson. In these accounts, the agreed codes were associated with relevant moments and episodes, with appropriate justification and analysis concerning the role of the trainee's content knowledge in the identified passages, with links to relevant literature.

The identification of these fine categories was a stepping stone with regard to our intention to offer a practical framework for use by ourselves, our colleagues and teacher-mentors, for reviewing mathematics teaching with trainees following lesson observation. A 17-point tick-list (like an annual car safety check) was not quite what was needed. Rather, the intended purpose demanded a more compact, readily-understood scheme which would serve to frame a coherent, content-focused discussion between teacher and observer. The key to the solution of our dilemma was the recognition of an association between elements of subsets of the 17 codes, enabling us to group them (again by negotiation in the team) into four broad, superordinate categories, which we have named (I) foundation (II) transformation (III) connection (IV)

contingency. These four units are the dimensions of what we call the ‘Knowledge Quartet’.

Each of the four dimensions is composed of a small number of subcategories that we judged, after extended discussions, to be of the same or a similar nature. An extended account to the research pathway described above is given in Rowland (2008a). The Knowledge Quartet (KQ) has now been extensively ‘road tested’ as a descriptive and analytical tool. As well as being re-applied to analytical accounts of the original data (the 24 lessons), it has been exposed to extensive ‘theoretical sampling’ (Glaser & Strauss, 1967) in the analysis of other mathematics lessons, in England and beyond (see e.g. Weston, Kleve & Rowland, 2013). As a consequence, three additional codes<sup>2</sup> have been added to the original 17, but in its broad conception, we have found the KQ to be comprehensive as a tool for thinking about the ways that content knowledge comes into play in the classroom. We have found that many moments or episodes within a lesson can be understood in terms of two or more of the four units; for example, a **contingent** response to a pupil’s suggestion might helpfully **connect** with ideas considered earlier. Furthermore, the application of content knowledge in the classroom always rests on **foundational** knowledge.

#### *Mathematical Knowledge for Teaching and the Knowledge Quartet*

It is useful to keep in mind how the KQ differs from the well-known Mathematical Knowledge for Teaching (MKT) egg-framework due to Deborah Ball and her colleagues at the University of Michigan, USA (Ball, Thames & Phelps, 2008). The Michigan research team refer to MKT as a “practice-based theory of knowledge for teaching” (Ball and Bass 2003, p. 5). The same description could be applied to the Knowledge Quartet, but while parallels can be drawn between the methods and some of the outcomes, the two theories look very different. In particular, the theory that emerges from the Michigan studies aims to unpick and clarify the formerly somewhat elusive and theoretically-undeveloped notions of SMK and PCK. In the Knowledge Quartet, however, the distinction between different *kinds* of mathematical knowledge is of lesser

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<sup>2</sup> These new codes, derived from applications of the KQ to classrooms within and beyond the UKs, are *teacher insight* (Contingency), *responding to the (un)availability of tools and resources* (Contingency) and *use of instructional materials* (Transformation) respectively.

significance than the classification of the situations in which mathematical knowledge surfaces in teaching. In this sense, the two theories are complementary, so that each has useful perspectives to offer to the other.

### ***Conceptualising the Knowledge Quartet***

The concise conceptualisation of the Knowledge Quartet which now follows draws on the extensive range of data referred to above. As we observed earlier, the practical application of the Knowledge Quartet depends as much on teachers and teacher educators understanding the broad characteristics of each of the four dimensions as on their recall of the contributory codes.

#### *Foundation*

Contributory codes: awareness of purpose; identifying errors; overt subject knowledge; theoretical underpinning of pedagogy; use of terminology; use of textbook; reliance on procedures.

The first member of the KQ is rooted in the foundation of the teacher's theoretical background and beliefs. It concerns their knowledge, understanding and ready recourse to what was learned at school, and at college/university, including initial teacher education, in preparation (intentionally or otherwise) for their role in the classroom. It differs from the other three units in the sense that it is about knowledge 'possessed', irrespective of whether it is being put to purposeful use. Both empirical and theoretical considerations have led us to the view that the other three units flow from a foundational underpinning.

A key feature of this category is its *propositional* form (Shulman, 1986). It is what teachers learn in their 'personal' education and in their 'training' (pre-service and inservice). We take the view that the possession of such knowledge has the potential to inform pedagogical choices and strategies in a fundamental way. By 'fundamental' we have in mind a rational, reasoned approach to decision-making that rests on something other than imitation or habit. The key components of this theoretical background are: knowledge and understanding of mathematics *per se*; knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics; and espoused beliefs about mathematics, including beliefs about why and how it is learnt.

### *Transformation*

Contributory codes: teacher demonstration; use of instructional materials; choice of representation; choice of examples.

The remaining three categories, unlike the first, refer to ways and contexts in which knowledge is brought to bear on the preparation and conduct of teaching. They focus on knowledge-in-action as *demonstrated* both in planning to teach and in the act of teaching itself. At the heart of the second member of the KQ, and acknowledged in the particular way that we name it, is Shulman's observation that the knowledge base for teaching is distinguished by " ... the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful" (1987, p. 15, emphasis added). As Shulman indicates, the presentation of ideas to learners entails their re-presentation (our hyphen) in the form of analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). Our second category, unlike the first, picks out behaviour that is directed towards a pupil (or a group of pupils), and which follows from deliberation and judgement informed by foundation knowledge. This category, as well as the first, is informed by particular kinds of literature, such as the teachers' handbooks of textbook series or in the articles and 'resources' pages of professional journals. Increasingly, in the UK, teachers look to the internet for 'bright ideas', and even for ready-made lesson plans. Teachers' choice and *use of examples* has emerged as a rich vein for reflection and critique (Rowland, 2008b). This includes the use of examples to assist concept formation, to demonstrate procedures, and the selection of exercise examples for student activity.

### *Connection*

Contributory codes: making connections between procedures; making connections between concepts; anticipation of complexity; decisions about sequencing; recognition of conceptual appropriateness.

The next category concerns the *coherence* of the planning or teaching displayed across an episode, lesson or series of lessons. Mathematics is notable for its coherence as a body of knowledge and as a field of enquiry. Indeed, a great deal of mathematics is held together by deductive reasoning. The pursuit of coherence and mathematical connections in mathematics pedagogy has been stimulated recently by the work of

Askew et al. (1997): of six case study teachers found to be highly effective, all but one gave evidence of a ‘connectionist’ orientation. The association between teaching effectiveness and a set of articulated beliefs of this kind lends a different perspective to the work of Ball (1990), who also strenuously argued for the importance of connected knowledge for teaching.

Our conception of coherence includes the *sequencing* of topics of instruction within and between lessons, including the ordering of tasks and exercises. To a significant extent, these reflect deliberations and choices entailing not only knowledge of structural connections within mathematics itself, but also awareness of the relative cognitive demands of different topics and tasks.

### *Contingency*

Contributory codes: responding to students’ ideas; deviation from agenda; teacher insight; (un)availability of resources.

Our final category concerns the teacher’s response to classroom events that were not anticipated in the planning. In some cases it is difficult to see how they could have been planned for, although that is a matter for debate. In commonplace language this dimension of the KQ is about the ability to ‘think on one’s feet’: it is about *contingent action*. Shulman (1987) proposes that most teaching begins from some form of ‘text’ – a textbook, a syllabus, ultimately a sequence of planned, intended actions to be carried out by the teacher and/or the students within a lesson or unit of some kind. Whilst the stimulus – the teacher’s intended actions – can be planned, the students’ responses can not.

Brown and Wragg (1993) suggested that ‘responding’ moves are the lynch pins of a lesson, important in the sequencing and structuring of a lesson, and observed that such interventions are some of the most difficult tactics for novice teachers to master. The quality of such responses is undoubtedly determined, at least in part, by the knowledge resource available to the teacher. For further details, see Rowland, Thwaites and Jared (2011).

In the following section, I shall illustrate the application of the KQ in the analysis of one primary mathematics lesson. The teacher, Sonia, was in the final stages of a one-year, graduate teacher education program in the UK.

## Primary mathematics teaching: the case of Sonia

### *Revised method*

In this phase of our classroom-based investigation, a trainee teacher was again videotaped teaching a lesson by one member of our research team, but our insights into the lesson were further enhanced as follows. Soon afterwards the team met to view the tape and to identify some key episodes in the lesson using the codes and categories developed in our earlier work. Later, one team member met with the trainee to view the videotape and to discuss some of these episodes. The interviewer drew the trainees' attention, one at a time, to key issues that had been identified by the team in their earlier analysis using the KQ, and invited the trainee to comment and offer their own perspective on the relevant episodes. We aimed to complete the three stages (videotaping the lesson, team reviewing the lesson, discussion with the trainee) in a short time span. In the case considered in this paper, the whole process occurred within one day.

We now consider three episodes from a lesson taught by Sonia, whose had previously majored in Religious Studies and Education. She joined the graduate primary teacher education program with concerns about her own mathematical knowledge and confidence. The lesson is with a Year 4 class (pupil age 8-9). She begins with a numerical task, as kind of 'warm-up', before introducing the learning outcome of the lesson - that pupils will be able to "... make and describe repeating patterns which involve translations and/or reflections".. We shall outline and discuss three episodes within the lesson.

### *Episode 1*

Sonia's beginning number activity involves finding complements in 100 and 1000. The three pairs of examples she uses are:

$$82 + ? = 100$$

$$35 + ? = 100$$

$$63 + ? = 100$$

$$820 + ? = 1000$$

$$350 + ? = 1000$$

$$630 + ? = 1000$$

A valuable insight into Sonia's ability to undertake subject knowledge **transformation** comes from her response - firstly instantaneous, then reflective - to the interviewer's question about the *choice of examples* that she uses for this activity.



Teachers often sequence their examples with the aim of making them progressively more demanding in some way as their students display success. But this raises the question of what makes one complement in 100 more demanding than another. More specifically, does Sonia have explicit (or implicit) decision criteria for her choices? In fact, Sonia's account, in the interview, was consistent with our own inference from observation:

- Interviewer: You know when you did these ... something add something equals...
- Sonia: mm
- Interviewer: ... 100 and 1000 and so forth, and the examples that you chose were 82 ...
- Sonia: Completely random.
- Sonia ... there was whatever came into my head.

It is tempting to suppose that since Sonia's 'choices' involved no apparent deliberation, they must have been arbitrary. Yet on further questioning, she was able to account for what had seemed to her to be 'random':

- Interviewer: ... Sometimes there's a choice, when you're giving examples, sometimes ... students or teachers have a particular reason for doing it. In your case these were just sort of...
- Sonia: What were they? There might have been a reason.
- Interviewer: 82, 35.
- Sonia: 35 because it was a smaller ... was an actually smaller number, I remember the reason for that one.
- Interviewer: So you had a smaller number after the ...
- Sonia: Yeah, after the big number. And then I made sure that that the ... the last digit of the 63 was a different last digit to the other two.
- Interviewer: Why did you have the smaller one ... in the middle?
- Sonia: Don't know. I just thought, this, I'd have a smaller number, like a substantially smaller number than 82.

And later:

- Sonia: The units ... the ten was intentional but the unit was random, in that case. And in the last one, 63, the ... ten was random but the unit was intentional.
- Interviewer: Right, so there's some ... thinking behind it.
- Sonia: Yeah.

Through this discussion, a rationale for Sonia's choice of examples has been teased out. She gives the impression that, although with help she is able to articulate her rationale, she was unaware of it in the moment (in the classroom) or until she was asked to talk about it. Ideally the examples that teachers use for pedagogical purposes should be

chosen deliberately, and with care (Rowland, 2008b). This type of discussion can be helpful in reflecting on practice and making explicit those decisions in planning, both actual and potential, that can affect the quality of children's learning experiences.

### *Episode 2*

Perhaps the most interesting episode arises when Sonia dwells on the pupils' solutions to  $63 + ? = 100$ . Since she is presented with three answers 37, 27 and 47 the way is open for **contingent action** on her part.

Instaed of simply identifying the correct answer, Sonia decides to invite a volunteer to discuss his method publicly. Matt firstly finds the complement in 10 of the 3 of 63, saying "If you do 3 add 7 that makes 10". It is at this point that Sonia prompts him by asking "Where have we got to?". There is something ambivalent in this utterance. In saying it, Sonia could simply be drawing the pupil back onto the task by asking him how much of the problem had been solved. On the other hand, the "where" could be a tacit way of suggesting a place in a specific mathematical sequence suggesting that she is guiding Matt into a sequential (or 'whole number') method. Either way he takes the cue, and increasing the 63 by 7 writes  $70 + 30 = 100$ . Sonia then ties things together asking "What have we added on?". She rings the 7 and the 30, asks the class what 30 and 7 make and finally draws out the answer to her original question:  $63 + 37 = 100$ .

In the earlier analysis of the videotape, the research team had made a conjecture (about supporting a sequential calculation strategy) concerning the intention of the question "Where have we got to?". This was tested in the interview, with illuminating consequences:

Interviewer: Yes, and he said, ... 3 add 7 that's 10, so, ... you ... referring back to what you said earlier on, you make it up to a nice number.

Sonia: Yes.

Interviewer: ... and you said, "Where have we got to?"

Sonia: Yes.

Interviewer: ... and he said 70.

Sonia: Yeah ... I think, was it 63, was the number?

Interviewer: Yes, but he said *three* add 7 is 10 ... were you trying to get him to do it in sequence, then?

Sonia: I thought that was what he was going to do, so I was just hoping he was, and tried to push him in that direction.

Interviewer: Yes, so "Where have you got to?" is just the right sort of prompt there ...

In this interchange Sonia confirms that her intention was to draw out a sequential process from Matt, even though his reference to 3, 7 and 10 may have derived from an intended split-tens strategy. This may also have helped to clarify the thinking of those children who gave answers of 27 and 47.

### *Episode 3*

The choices of shapes Sonia selects to transform in the next stage of her lesson reveals some shortcomings in her **foundation** knowledge. In particular, she does not appear to realise that the internal properties of a transformed figure can mask certain effects of a transformation, particularly reflection.

With the learning objective of pattern-making in place, she tries to establish that when a shape is transformed, a *second* shape is generated which is “the same” as the first. This is somewhat confusing, because if her notion of a transformation is a movement, when using *objects*, there will not be two shapes. The moved object will become the *image* of the transformation but the domain shape (the pre-image or ‘argument’ of the transformation) will no longer exist. Of course, this dilemma does not arise if the transformation is seen, not as a movement, but as a relationship between *pairs* of points (and in consequence between pairs of shapes) in the plane.

The shapes (including a circle and a rectangle) that she ‘chooses’ to use for demonstration are both have a high degree of symmetry, and for this reason they do not reveal a change of orientation under the transformations of reflection and translation. This points to some weak **transformational** thinking (in the sense of the knowledge quartet!) on Sonia’s part. The circle is spectacularly ineffective in conveying the particular properties of a translation. If a circle  $C$  has translation image  $C'$ , then  $C'$  is also the image of  $C$  under a reflection or a rotation. An astute pupil presents her with an opportunity for **contingent action**. This pupil perceives the pedagogical inadequacy of these symmetrical shapes, and offers “If you reflect it with an L shape it wouldn’t turn out the same”. This time, whilst Sonia endorses the pupil’s response she makes no attempt to enact his suggestion publicly. However, when questioned later the same day Sonia readily saw this as a missed opportunity:

Interviewer: ... it’s about the boy who did the ... who asked for the L-shape.

Sonia: Yes.

Interviewer: The shapes that *you* chose were a rectangle.  
 Sonia: Yes.  
 Interviewer: ... and a circle, which ... have got a certain amount of regularity.  
 Sonia: Yeah.  
 Interviewer: ... if you flip the ... the rectangle, the same ... but the L-shape ... hasn't got any symmetry in it, if you like.  
 Sonia: mm  
 Interviewer: Emm, so did you, were you aware of that, or just ...  
 Sonia: [laughs] I took random shapes off a pile [laughs] um, yes.  
 Interviewer: Well, you can see the boy's point ...  
 Sonia: Oh yes, definitely.  
 Interviewer: It's quite a good reply.  
 Sonia: If I were to do it again, I would ...  
 Interviewer: It would be striking what has happened to the shape if itself it didn't have any symmetry.  
 Sonia: It would be much easier for them to see.

So here we hope that the discussion has helped Sonia extend her understanding of these transformations, and how particular example shapes can be used to demonstrate the essence of a specific transformation more effectively than others. By reflecting on selected aspects of the mathematical content of her teaching, Sonia is identifying areas of her mathematics content knowledge - both SMK and PCK - where there is scope for development. She is also becoming aware of areas where she had good instincts which she might now incorporate into rational decision-making.

### **Supporting research and teaching development**

The KQ has found two intersecting user groups since its emergence a decade ago. In this section, we outline resources developed to support these user groups.

#### ***Teacher education and teaching development***

As we remarked earlier, one of the goals of our original 2002 research was to develop an empirically-based conceptual framework for mathematics lesson review discussions with a focus on the *mathematics content* of the lesson and the role of the trainee's mathematics subject matter knowledge (SMK) and pedagogical content knowledge (PCK). In addition to the kind of 'knowledgeable-other' analysis and formative feedback exemplified in the cases of Sonia in this paper, it has also been used to support teachers wanting to develop their teaching by means of reflective evaluation on their own classroom practice (Turner, 2012; Corcoran, 2011). Specifically, the KQ is a tool which enables teachers to focus reflection on the mathematical content of their teaching.

However, both teacher educators and teachers must first learn about the tool, and how to put it to good use. A book (Rowland, Turner, Thwaites & Huckstep, 2009) was written to address the needs of this user-group, especially in relation to primary mathematics. It describes the research-based origins of the KQ, with detailed accounts of the four dimensions, and separate chapters on key codes such as Choice of Examples. The narrative of the book is woven around accounts of over 30 episodes from actual mathematics lessons. We return to this use of the KQ towards the end of this paper.

### ***Observational research into mathematics teaching***

In some respects, the needs of researchers using the KQ as a theoretical framework for lesson analysis are the same as those of teachers educators, but they are different in others. In particular, a broad-brush approach to the four KQ dimensions often suffices in the teacher education context, and may even be preferable to detailed reference to constituent codes. For example, identifying Contingent moments and actual or possible responses to them need not entail analysis of the particular triggers of such unexpected events. On the other hand, reflections or projections on Transformation usually involve reference to examples and representations. Our writing about the KQ (e.g. Rowland et al., 2005) initially focused on explaining the essence of each of the four dimensions rather than identifying definitions of each of the underlying codes. However, a detailed KQ- analysis of a record (ideally video) of instruction necessarily involves labelling events at the level of individual KQ-codes, prior to synthesis at dimension level (Foundation, Transformation etc). This, in turn, raises reliability issues: the coder needs a deep understanding of what is intended by each code, going beyond any idiosyncratic connotations associated with its name. Addressing this issue, a Cambridge colleague of ours wrote as follows:

Essentially, the Knowledge Quartet provides a repertoire of ideal types that provide a heuristic to guide attention to, and analysis of, mathematical knowledge-in-use within teaching. However, whereas the basic codes of the taxonomy are clearly grounded in *prototypical teaching actions*, their grouping to form a more discursive set of superordinate categories – Foundation, Transformation, Connection and Contingency – appears to risk introducing too great an interpretative flexibility *unless these categories remain firmly anchored in grounded exemplars of the subordinate codes*” (Ruthven, 2011, p.85, emphasis added).

In 2010 a Norwegian doctoral student wrote to us as follows: “I need a more detailed description on the contributory codes to be able to use them in my work. Do you have a

coding manual that I can look at?”. This enquiry, Ruthven’s comment, and our growing sense of the risk of “interpretive flexibility” led us to initiate a project to develop an online coding manual, with the needs of researchers particularly in mind.

The aim of the project was to assist researchers interested in analysing classroom teaching using the Knowledge Quartet by providing a comprehensive collection of “grounded exemplars” of the 20 contributory codes from primary and secondary classrooms. An international team of 15 researchers was assembled. All team members were familiar with the KQ and had used it in their own research as a framework with which to observe, code, comment on and/or evaluate primary and secondary mathematics teaching across various countries, curricula, and approaches to teaching. The team included representatives from the UK, Norway, Ireland, Italy, Cyprus, Turkey and the United States. In Autumn 2011 team members individually scrutinised their data and identified prototypical classroom-exemplars of some of the KQ codes. To begin with, a written account of each selected classroom *scenario* was drafted. Often this included excerpts of transcripts and/or photographs from the lesson. Then a *commentary* was written, which analysed the excerpt, explaining why it is representative of the particular code, and why it is a strong example. Each team member submitted scenarios and commentary for at least three codes from his/her data to offer as especially strong, paradigmatic exemplars. In March 2012, 12 team members gathered in Cambridge, and worked together for two days. Groups of three team members evaluated and revised each scenario and commentary. The scenarios and commentaries were then revised, on the basis of the conference feedback. Further details of the participants and methodology are given in Weston, Kleve & Rowland (2013).

These scenarios and commentaries now combine to form a “KQ coding manual” for researchers to use. This is a collection of primary and secondary classroom vignettes, with episodes and commentaries provided for each code. The collection of codes and commentaries is now freely available online at [www.knowledgequartet.org](http://www.knowledgequartet.org). At the time of writing, the website is ‘live’ but subject to further development. We encourage researchers and teacher educator to use and share this website in the cause of improved clarity about what each of the KQ codes ‘looks like’ in a classroom setting.

## **Conclusion**

Mathematics teaching is a highly complex activity; this complexity ought to be acknowledged when teaching is analysed and discussed, and due attention given to discipline-specific aspects of pedagogical decision and actions, beyond generic aspects of the management of learning. Strong, clear conceptual frameworks assist in the management of this complexity. By attending to events enacted and observed in actual classrooms, with a specific focus on the subject-matter under consideration, the KQ offers practitioners and researchers such a conceptual framework, particularly suited to understanding the contribution of teacher knowledge to mathematics teaching. For practitioners and teacher educators, the KQ is a tool for identifying opportunities and possibilities for teaching development, through the enhancement of teacher knowledge, as indicated, for example, in the book Rowland et al. (2009). Especially in the case of pre-service teacher education, it is beneficial to limit the post-observation review meeting to one or two lesson fragments, and also to only one or two dimensions of the KQ, in order to focus the analysis and avoid overloading the trainee-teacher with action points.

In this paper I have emphasised the progression from observation of teaching to its description and analysis, but I have been less explicit about the evaluation of teaching. In the spirit of reflective practice, the most important evaluation must be that of the teacher him/herself. However, this self-evaluation is usefully provoked and assisted by a colleague or mentor, using the KQ to identify a small number of tightly-focused discussion points to be raised in a post-observation review. We have suggested that these points be framed in a relatively neutral way, such as “Could you tell me why you ... ?” or “What were you thinking when ... ?”. It would be naïve, however, to suggest that the mentor, or teacher educator, makes no evaluation of what they observe. Indeed, the observer’s evaluation is likely to be a key factor in the identification and prioritisation of the discussion points. In post-observation review, it is expected that the ‘more knowledgeable other’ will indicate what the novice did well, what they did not do and might have, and what they might have done differently. The KQ is a framework to organise such evaluative comments, and to identify ways of learning from them.

The KQ has been successfully applied across different phases of schooling, and in diverse cultures, but we mention, in conclusion, a development that we had not

originally anticipated. Having attended presentations about the KQ in cross-disciplinary settings, some teacher education colleagues working in subjects other than mathematics – such as language arts, science and modern foreign languages education – have seen potential in the KQ for their own lesson observations and review meetings. They sometimes ask whether they could adapt and adopt the KQ for their own purposes. This raises the issue: can a framework for knowledge-in-teaching developed in one subject discipline be legitimately adopted in another? My reply usually begins as follows: what might the conceptualisations of the dimensions of the KQ, beginning with Foundation, look like in this other discipline? An answer to this question could set the scene for empirical testing of the KQ in another subject area.

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An extended version of this paper, featuring case-analyses of two different mathematics lessons, has been submitted to the *Sisyphus – Journal of Education*, an editorial initiative of the Institute of Education of the University of Lisbon.

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